

MATHEMATICAL MODEL FOR IONIZATION WAVES IN A PLASMA
PRODUCED BY RADIO FREQUENCY IN A MAGNETIC MIRROR.

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ABSTRACT.

By using Langmuir probes, in a Argon plasma produced by RF electric field, we have detected large spatial variations of density and temperature along the axis of symmetry of the system [01]. These spatial variations have characteristics similar to those Striations waves found in DC glow discharges, in the presence of a constant DC electric field.

In this work, we present a mathematical model, developed to explain Striations waves phenomena, taking into consideration Striations waves generated by a DC oscillating electric field in a plasma produced by RF electric field. From this mathematical model is gotten the averages values of plasma potential amplitude V_p^{theor} and the oscillating electric field amplitude E_{osc}^{theor} and the same are compared with those obtained experimentally.

I - INTRODUCTION

We have built a system of Multiple Mirror with a chain of five simple mirrors [01]. We have produced Argon plasma by RF field and observed Striation or Ionization Waves (I.W.) in the regime of glow discharge [01]. Striations were originally observed in a low pressured DC glow discharge, with the electric field as in [02,03,04]. The principal result of those observations were that the product of the DC electric field E , used to generate the DC plasma, by the oscillation wavelength λ obtained from the plasma density profile was a constant named Novak's constant V_λ , ie, $E \cdot \lambda = V_\lambda$. In the reference [01], is reported the experimental observation of I.W. in a multiple mirror system, in a plasma produced by R.F. field, where the characteristics of I.W. are depending on an DC oscillating electric E field. In contrast to a usual Striation formation [02,03,04], that device [01] produced the same phenomena apparently without the need of a constant DC electric field and a DC glow discharge.

In section II a simplified mathematical model is developed to explain the I.W. taking into consideration a plasma produced by a RF field and the I.W. phenomena produced by an DC oscillating electric field, and compare the results with the experimental data reported in [01]. In particular, the theoretical average values of plasma potential amplitude V_p^{theor} , and the oscillating electric field amplitude E_{osc}^{theor} compare favorably with the experimental data. In section III we give the conclusion.

II - MATHEMATICAL MODEL FOR IONIZATION
WAVES PRODUCED BY RADIO FREQUENCY (RF)

In the early of seventy years, I. Grabec published a mathematical model about the phenomena Ionization Waves (I.W.) for a DC glow discharge. In that, I.W. was described as a no-linear phenomenon, where a set of equation describing a weakly ionized plasma was got from hydrodynamics and Poisson equations [05].

The mathematical model proposed in this work is based on Grabec's Model, published in reference [05]. The principal difference between our work and Grabec's model is that we do not take into consideration the Poisson's equation (in our plasma $n_e \approx n_i$). Furthermore, our plasma is produced by RF electric field, instead a constant DC electric field as in the previous model, and the I.W. presented in this paper are generated by a DC oscillating electric field (cf equation II-07) instead a constant DC electric field [02,03,04]. The ionization frequency is (cf equation II-04), in this model, a square root function of the temperature, while, in the Grabec's model, the ionization frequency is a exponential function of temperature. Finally, in the equations II-01 and II-02, gotten from Grabec's model, we are taking out the temporal dependence, since our I.W. phenomena is stationary one. An other difference from previous model, is that in this model we are supposing that the

electron density have a sinusoidal variation (cf equation II-05 and figure II-1). The same approximation was done for the DC oscillating electric field electric field that generates the I.W. phenomena.

We know that ionization waves (I.W.) always take place in the presence of an electric field, but this electric field do not need be a constant DC electric field how is the D.C. glow discharge case [02,03,04]. Our experimental results were obtained from a plasma produced by RF field, and the characteristics of the I.W. depends of an DC oscillating electric field [01]. To explain the experimental observation of the spatial variation of the electron density, figure II.1, and plasma potential fluctuation (cf figure IV of reference [01]), a numerical model was developed taking into consideration that I.W. were generated by a DC oscillating electric field in a plasma produced by RF electric field. The figure II.2 shows the assumed profiles of I.W. of a plasma produced by RF field, and, as we will show, they are in agreement with our mathematical model proposal.

It is important to make clear that, by using this mathematical model, we will get theoretical average values of plasma potential amplitude v_p^{theor} and oscillating electric field amplitude E_{osc}^{theor} , and then, compare these results with those correspondents average experimental amplitudes values, ie, we will not reproduce theoretically the experimental profiles of I.W. obtained experimentally (cf figure II-1), but only get the average theoretical amplitudes values and compare it with experimental one.

Taking into consideration the following set of equations, with the conditions of constant flow, quasi-neutrality and stationary ionization waves, we have:

$$\partial y / \partial x + n_i / t_a = n_i Z \text{ (equation of continuity)} \tag{II-01}$$

$$\partial (y T_u) / \partial x = - e n_e v_{xe} E - n_e T_u / t_1 - n_e Z V_i e \text{ (equation of energy density)} \tag{II-02}$$

$$y = n_i v_{xi} = n_e v_{xe} = - D_a \partial n / \partial x \text{ (equation of flow conservation)} \tag{II-03}$$

$$Z = N_o \sigma v_{oe} \text{ (ionization frequency equation)} \tag{II-04}$$

The equations II-01 and II-02 were gotten from reference [05], but taking out the temporal dependence. Since from our experimental conditions we have ambipolar diffusion, we got the equation II-03 from reference [07]. The ionization frequency Z, equation II-04, was gotten from [06]. v_{oei} is the thermal velocity of electrons and ions, N_n is the density of neutral particles, σ is the ionization cross section of argon by electron impact and v_{xei} is the flow velocity of electron and ions.

To verify a internal consistency of the equations II-01 to II-04 with the ionization waves, we will suppose that the experimental electron/ion density profiles have the following sinusoidal variation:

$$n = n_o + n_1 \cos(2\pi x / \lambda) \tag{II-05}$$

where n_o and λ are the average electron density and the experimental average oscillation wavelength, respectively, got from figure II-1. n_1 is an adjustment factor, that will be calculated later, by using the figure II-1.

The figure II-2.a shows the proposed profile of the spatial variation of the electron density, in according with equation II-05. The symbol n_o and λ in this figure are the same of the figure II-1. Therefore, the figure II-2.a is a model for the spatial variation of the electron density detected experimentally along the axis of symmetry of the system, (cf figure II-1). T_u is the electron energy $3K_B T_e / 2$ where T_e is the electron temperature obtained experimentally [01].

From equations II-01 and II-02, we have for $n_e \approx n_i$:

$$E y = - n [(1/t_1 - 1/t_a) T_u / e + Z (T_u / e + V_i)] \tag{II-06}$$

where n is given by the equation II-05, V_i is the ionization potential of Argon (A_r^+), t_a and t_1 are the relaxation time due ambipolar diffusion and elastic collision respectively.

Since our experimental result were got from a plasma produced by a RF electric field [01], and our I.W. phenomena is assumed to be generated by an oscillating DC electric field instead of a

constant DC electric field, we may represent this electric field E by the following sinusoidal variation:

$$E = E_{osc}^{theor} \sin(2\pi x/\lambda) \quad (II-07)$$

where E_{osc}^{theor} is the oscillating amplitude of the electric field. The figure II.2.c shows the proposed profile of the spatial variation of the DC oscillating electric field, in according with equation II-07.

From equations II-03 and II-05, we get:

$$y = (2\pi D_a n_1 / \lambda) \sin(2\pi x / \lambda) \quad (II-08)$$

Making the product of the equation II-07 by the II-08, and obtaining its average value $\langle E.y \rangle$, we have

$$\langle E.y \rangle = (E_{osc}^{theor} \pi D_a n_1 / \lambda) \quad (II-09)$$

Making the result of equation II-09 equal to the average value $\langle E.y \rangle$ of equation II-06, we get

$$\langle E_{osc}^{theor} \rangle = - (\lambda n_0 / \pi n_1 D_a) [Z(\bar{T}_u / e + V_i) + (1/t_1 - 1/t_a) \bar{T}_u / e] \quad (II-10)$$

Using the average value of \bar{T}_u obtained experimentally, cf table I, we may calculate the values of the ionization frequency Z , equation II-04. For the values corresponding to the (*) in table I of this work ($\bar{T}_e = 10$ eV, therefore $\bar{T}_u = (3/2)K_B \bar{T}_e = 15$ eV), the ionization cross section of Argon by electron impact [08] is between:

$$\sigma = (0.025, 0.08) \pi a_0^2 \quad (II-11)$$

where a_0 is the Bohr's radius.

Since the value of pressure used experimentally was 3.0 mTorr, the value of the neutral density Argon particles is $N_0 = 9.9 \cdot 10^{13} \text{ cm}^{-3}$ ($T = 300^\circ \text{K}$ is room temperature). Therefore, the values of ionization frequency Z are between:

$$Z = (4.7, 15.) \cdot 10^4 \text{ (s}^{-1}\text{)} \quad (II-12)$$

From equations II-11 and II-12 the equation II-10 came out to:

$$|E_{osc}^{theor}| \approx |-122 n_0 / n_1| \text{ (V/cm)} \quad (II-13)$$

where n_0 is the experimental average electron density, cf figure II-1 in

this paper, or table I of reference [01], and n_1 is the adjustment factor of the equation II-05. In this work $n_1 \approx n_0/2$, therefore

$$|E_{osc}^{theor}| \approx 240 \text{ V/cm}$$

We have also used the following quantities to get numerical value for the equation II-10:

R	=	mean radius of pyrex tube = 3.5 cm
D_i	=	diffusion coefficient of ions $\approx 5.9 \cdot 10^4 \text{ cm}^2/\text{s}$
D_e	=	diffusion coefficient of electrons $\approx 5.9 \cdot 10^8 \text{ cm}^2/\text{s}$
$L_{e,n}$	=	mean free path of electrons = $1/\sigma_e N_0 \approx 7.4 \text{ cm}$
σ_e	=	cross sections of electron $1.4 \cdot 10^{-15} \text{ cm}^2$
$L_{i,n}$	=	mean free path of ions = 1.7 cm
σ_i	=	cross section of ions = $6.0 \cdot 10^{-15} \text{ cm}^2$
V_{ee}	=	thermal velocity of electrons = $1.9 \cdot 10^8 \text{ cm/s}$ (for $\bar{T}_e = 10$ eV, corresponding to the value got from the (*) in table I).
V_{ei}	=	thermal velocity of ions = $1.0 \cdot 10^5 \text{ cm/s}$, where the average ion temperature T_i is from 0.2 to 0.3 eV.
t_1	=	relaxation time due elastic collision = $L_{e,n}/V_{ee} = 3.9 \cdot 10^{-8} \text{ s}$
t_a	=	ambipolar relaxation time = $R^2/D_a = 4.1 \cdot 10^{-6} \text{ s}$
D_a	=	ambipolar diffusion coefficient = $3.0 \cdot 10^6 \text{ cm}^2/\text{s}$

It is important to make clear, that to get numerical values for the equation II-10, (cf equation II-13), we have used the average value of electron temperature corresponding to the (*) of table I, and the average value of density, figure II-1. To obtain the other values of the theoretical oscillation amplitude of the electric E_{osc}^{theor} field, shown in table I (cf equation II-10), we need the others values of experimental oscillation amplitude V_{ep}^{exp} of the plasma potential, as well \bar{T}_e or λ_{exp} , got from others

experimental profiles that will not follow in this work, because they are similar as that of figure II-1. About the experimental oscillation amplitude v_{osc}^{exp} of the plasma potential, showed by the (*) in table I, it corresponds to the value obtained from figure IV of reference [01].

The theoretical result of the average oscillating electric E_{osc}^{theor} field amplitude, obtained from equation II-10 will be compared with that average experimental values of the oscillating electric E_{osc}^{exp} field amplitude, got from

figure IV of the reference [01], (cf also the table I of this work). In this case we are assuming that $n_0 = n_1/2$ in the equation II-13, ie, the average value of the electron n_0 density obtained from figure II-1 is about $n_0 = 6.0 \times 10^8 \text{ cm}^{-3}$, and since from equation II-05, we have by using $\cos(2\pi x/\lambda) = -1, 0, 1$, the following values for the electron density:

- a) $n = n_0 - n_1 = n_{min}$ = minimum electron density,
 - b) $n = n_0$ = average electron density,
 - c) $n = n_0 + n_1 = n_{max}$ = maximum electron density,
- and, from our experimental result, figure II-1, can be seen that $n_{min}/n_{max} \approx 9.0 \times 10^8 / 3. \times 10^8$, therefore, from a) and c) above, we got $n_1 \approx n_0/2$.

For $n_1 = n_0/2$ in the equation II-13, we get $E_{osc}^{theor} \approx 240$ volts. The table I of this paper, shows the theoretical, E_{osc}^{theor} , and experimental values, E_{osc}^{exp} , of oscillating electric field amplitude.

In all this paper we are assuming $n_1 \approx n_0/2$ for all results of table I, because $n_1 \approx n_0/2$ was the result obtained in all experimental condition [01]

Using equation II-07 we may calculate the theoretical value of the plasma potential v_p^{theor} , ie,

$$E = E_{osc}^{theor} \sin(2\pi x/\lambda) = -v_x v_p \quad (II-14)$$

$$v_p = (E_{osc}^{theor} \lambda / 2\pi) \cos(2\pi x/\lambda) \quad (II-15)$$

where v_p represents the spatial variation of the plasma potential observed experimentally (cf figure IV of reference

[01]), in according to our proposed model, cf figure II.2.b.

From the equation II-15 we may get the theoretical amplitude oscillation of the plasma potential v_p^{theor} , ie,

$$v_p^{theor} = (E_{osc}^{theor} \lambda / 2\pi) = 240 (3/2\pi) = 115 \text{ V} \quad (II-16)$$

where $\lambda = \lambda_{exp}$ is the experimental average oscillation wavelength corresponding to the (*) of table I (cf figure II-1).

It is important to note that, to get numerical value for the equation II-16 ($v_p^{theor} \approx 115 \text{ V}$) we have only used the values corresponding to the (*) in table I. The averages values of electron density n_0 and oscillation wavelength λ were got from figure II-1. Furthermore, the value of $E_{osc}^{exp} = 2\pi v_p^{exp} / \lambda_{exp}$ in table I, were got from the figure IV of reference [01].

Finally, the table II shows the theoretical, v_p^{theor} and experimental v_p^{exp} , values of the oscillation amplitude of the plasma potential. It is important to note that we have only used the values corresponding to the (*) of table I and figure II-1. To obtain the other values of the theoretical oscillating amplitude of the electric field and the theoretical oscillating amplitude of the plasma potential, shown in table II, the work is similar to this one, and will not follow in this paper.

CONCLUSION

In this work we presented a mathematical model, developed to explain striations waves phenomena, taking into consideration a plasma generated by radio frequency waves, and the I.W. phenomena produced by an DC oscillating electric field, therefore, different from Grabec's model [05] where a DC glow discharge and the I.W. were produced by a constant DC electric field.

The results obtained from this mathematical model, are in agreement with those obtained experimentally, ie, the theoretical average values of the DC oscillating electric field amplitude E_{osc}^{theor} and the theoretical average values of the oscillating plasma potential v_p^{theor} amplitude predicted by the theory are in agreement with the experimental ones, cf table II. Furthermore, the theoretical profiles assumed for the ionization waves shown in figure II.2, are in agreement with the theoretical model developed.

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TABLE I - EXPERIMENTAL AND THEORETICAL VALUES OF THE OSCILLATING ELECTRIC FIELD AMPLITUDE.

\bar{T}_e (eV)	λ_{exp}	v_p^{exp} (V)	E_{osc}^{exp} (V/cm)	E_{osc}^{theor}
10.0 *	3.0 *	95.± 5.*	200.± 23.	230.
8.0	3.8	90.± 4.	150.± 15.	200.
7.0	2.7	55.± 4.	130.± 19.	150.
11.0	3.0	100.± 5.	210.± 25.	240.

Where $v_p^{exp} = (\Delta v_{p1} + \Delta v_{p2})/2$ is the experimental oscillating amplitude of the plasma potential, obtained from figure IV of reference [01]. The experimental oscillating electric field amplitude were obtained from the relation $E_{osc}^{exp} = 2\pi v_p^{exp} / \lambda_{exp}$, while the theoretical oscillating electric field amplitude E_{osc}^{theor} were obtained from equation II-10. λ_{exp} is the experimental average oscillation wavelength, cf figure II-1. \bar{T}_e is the experimental average electron temperature [01]. The (*) in this table, corresponds to the experimental average values used to get numerical values for equations II-10 and II-16. The others values of \bar{T}_e and n_0 comes from reference [01].

TABLE II - EXPERIMENTAL AND THEORETICAL VALUES OF THE OSCILLATING PLASMA POTENTIAL AND ELECTRIC FIELD AMPLITUDES.

THEORETICAL DATAS		EXPERIMENTAL DATAS	
v_p^{theor} (V)	E_{osc}^{theor} (V/cm)	v_p^{exp} (V)	E_{osc}^{exp} (V/cm)
110.	230.	95.± 5.	200.± 23.
120.	200.	90.± 4.	150.± 15.
65.	150.	55.± 4.	130.± 19.
115.	240.	100.± 5.	210.± 25.

Where v_p^{theor} is the theoretical oscillating amplitude of the plasma potential, cf equation II-16. The theoretical, E_{osc}^{theor} and experimental E_{osc}^{exp} , oscillating electric field amplitude, and the experimental oscillating amplitude of the plasma potential v_p^{exp} , came from table I.

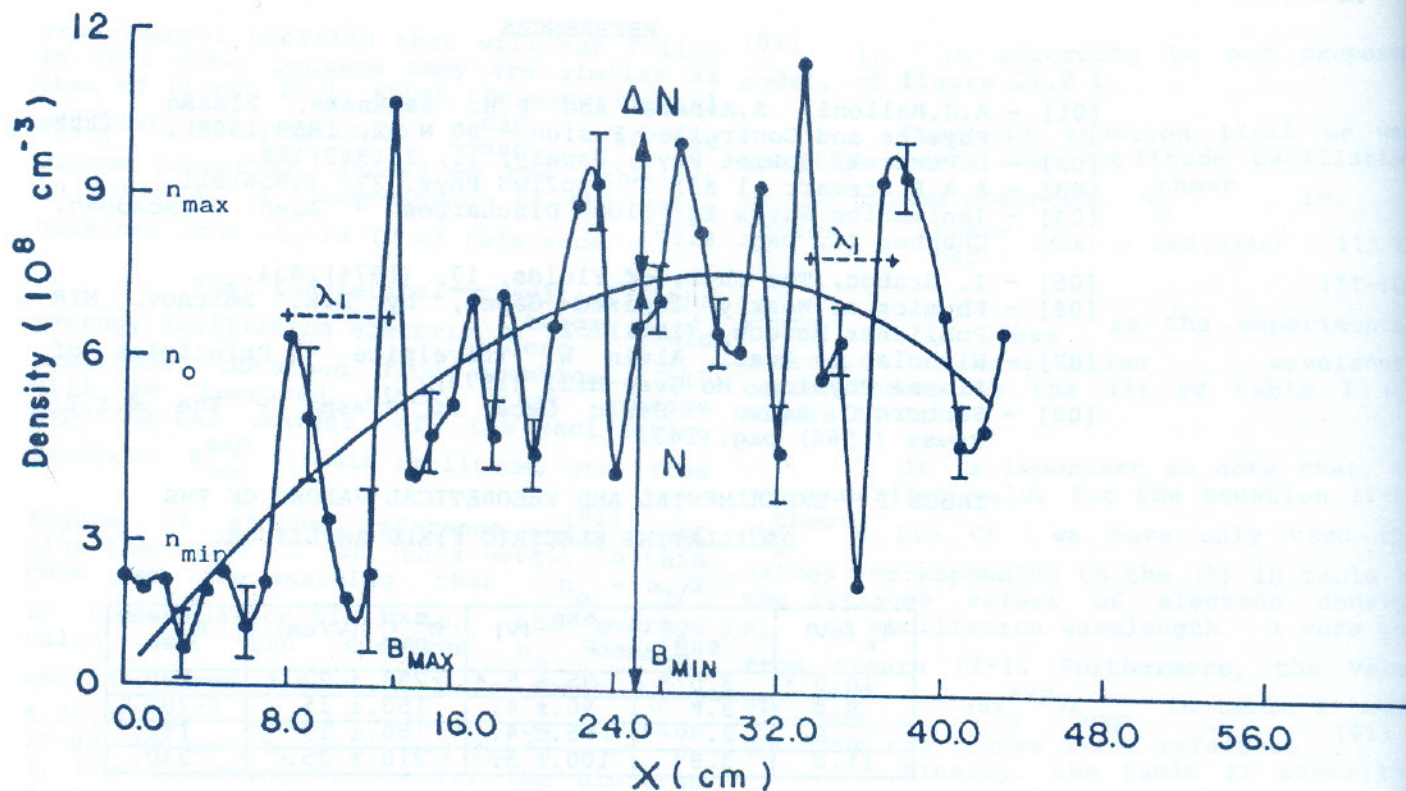


FIG. II-1 - Density along the axis obtained from a plasma produced by RF field (01). n_0 , n_{max} , n_{min} are the experimental values of average, maximum and minimum electron density, respectively.

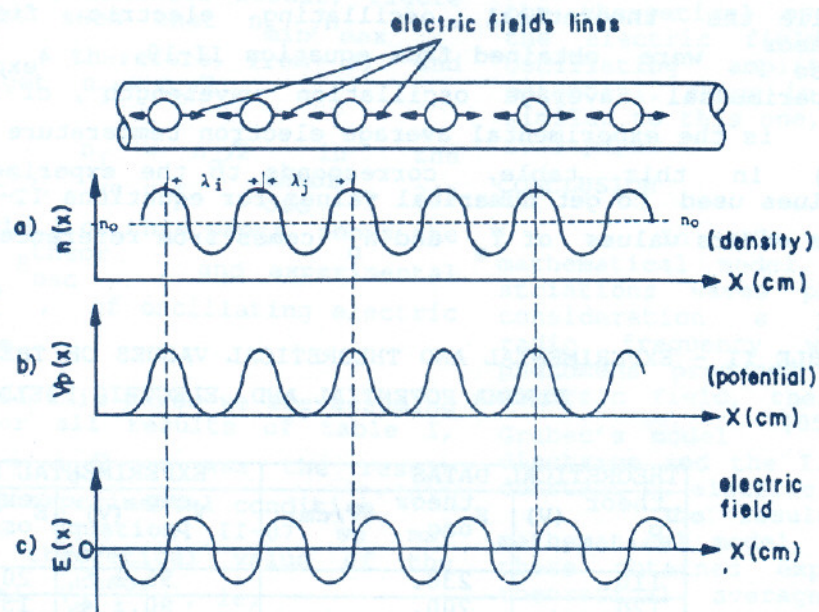


FIG. II-2 - Profiles of ionization waves (striations) for a plasma produced by RF field (without current). According to our model, the electrical field is an oscillating function instead of a constant.

$\lambda_{i,j}$ Is the same of figure II-1. In this figure $\lambda_i = \lambda_j = \lambda_{exp}$

n_0 Is the experimental average electron density, cf. figure II-1.