# PRESSURE BEHAVIOR ANALYSIS ON A HOLLOW CATHODE PLASMA ETCHING SYSTEM

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## ABSTRACT

Important questions on a plasma etching system arises from gas flow issues. This article represents a first step to obtain a thorough understanding of the gas behaviour on a Hollow Cathode Plasma Etching system. According to this objective, the pressure performance within a Plasma chamber, and on a remote positions of the gas feed line were analysed. Experimental measurements were compared to the predictions originated from an analytic model. Excellent agreement between theoretical and experimental values was achieved.

#### 1. INTRODUCTION

In the study of plasma processes vacuum technology is largely secondary in that it is part of other technologies that are central to analysis, research, development, and manufacturing [1]. Even though, it is of fundamental relevance on the plasma material processing. That is, In order to achieve a comprehensive description of the plasma phenomena it is paramount to develop, at first place, a solid understand of the gas phase behavior [2]. Aiming this goal, this article proposes a model for the pressure performance within the plasma vessel. This analysis employs the basic vacuum technology equations to obtain a set of formulas valid for the device in focus.

Currently, a series of papers struggles to develop the space profile of electrons and ions densities inside a plasma environment [3-4]. Nonetheless, these approaches disregard the influence of the gas profile. In general, they consider the gas pressure homogeneous through the entire vessel. It is reasonable to proceed this way on large chamber, but it becomes inconvenient with respect to more complex structures as Hollow Cathode sources [5]. For the later is advisable to verify the contribution of the gas flow pattern to the plasma dynamic. Aware of this aspect, this article initiates a study of the gas phase in a plasma discharge, in order to analyze its influence on the plasma behavior.

In this work we measure the dependence of the pressure inside a plasma chamber and in a remote position upstream the gas feed line, to the gas throughput into the system. In addition, an analytical model of the vacuum system, in terms of a differential equation system, was developed and it solution was compared with numerical calculations, based on a solution of the differential equation system through the

## 2. EXPERIMENTAL

A schematic diagram of the experimental apparatus is shown in Figure 1. This apparatus consists of a cylindrical chamber with 170 mm radius and 400 mm long, connected to a gas feed line at one side and to a vacuum pump to the other. One pressure gauge is directly attached to the chamber and another pressure gauge is connected to the gas line, through a small volume, upstream to the gas line. This configuration aim to estimate the pressure inside a Hollow Cathode to be connected inside the chamber in future experiments as already reported in previous articles [6]. Its presence on this work act as a support for further studies where the Hollow Cathode Chamber will be analyzed.

The purpose of the small volume, of 30 mm radius and 50 mm long, is to aloud the physical adjustment between the pressure measurement device and the gas line pipe. Prior to Ar feed the vacuum chamber is pumped down to a pressure below  $10^{-6}$  torr ( $10^{-4}$  Pa) using a combination of a roots and a mechanical pump, providing both an effective pumping speed of approximately 100 L/s ( $0.1 \text{ m}^3$ /s). The argon gas was inserted in the chamber on a throughput range of 1 - 100 sccm, corresponding to a pressure range of 0.04 - 15 mtorr (0.005 - 2 Pa) on the vacuum chamber, and to a pressure range of 8 - 80 mtorr (1 - 10 Pa) on the small volume.



Figure 1 – Schematic diagram of experimental setup, where the principal vacuum system components are show.

In order to assess the accuracy of the measured results an analytical model of this vacuum system was derived. With the support of this model, a fit of the experimental data was obtained. This model and the data analysis are presented on the next sections.

Runge-Kutta method. Both results, analytical and numerical were compared to the experimental measurements.

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#### 3. SYSTEM MODEL

In order to obtain a mathematical treatment of the problem, a model as schematically demonstrated in Figure 2 was developed. To build up the model, the small volume used to promote the physical connection between the pressure gauge and the gas line was considered as being a second chamber linked to the plasma chamber through a long round tube.



Figure 2 – Analog system to the experimental setup diagrammatically exposed in Figure 1.

Following this scheme there will be two differential equations to describe the pressure performance: one to each chamber [1]. These equations can be gathered on the differential equation system below,

$$\begin{cases} \frac{dP_1}{dt} = -\left(\frac{C'}{2V_1}\right)P_1^2 + \left(\frac{C'}{2V_1}\right)P_2^2 + \left(\frac{Q_1}{V_1}\right) \\ \frac{dP_2}{dt} = \left(\frac{C'}{2V_2}\right)P_1^2 - \left(\frac{C'}{2V_2}\right)P_2^2 - \left(\frac{S_p}{V_2}\right)P_2 \end{cases}$$
(1)

where  $P_1$  represents the pressure inside the small volume,  $V_1$  is its volume,  $P_2$  the pressure inside the plasma chamber,  $V_2$  is the volume of this chamber,  $Q_1$  is the gas throughput to the vacuum system,  $S_p$  is the vacuum pump speed, and t is the time considered in the calculations. The parameter  $C^c$  in the equation (1) is the amount of the tube conductance that doesn't change with the pressure, that is [1],

$$C' = \frac{\pi d^4}{128nl} \tag{2}$$

where *d* represents the tube diameter, *l* is the tube long, and  $\eta$  corresponds to the gas viscosity. The differential equations in (1) govern the pressure behavior inside two chambers connected through a pipe and are to be solved simultaneously to obtain the pressure dependence on the gas throughput in steady state.

### **3.1. ANALYTICAL SOLUTION**

The ordinary differential equation (ODE) system in (1) doesn't possess a closed form solution. Nonetheless, the pressure ratio  $P_1/P_2$  can reach values of even to the fourth order of magnitude in steady-state. On this way it is reasonable to consider  $P_1 >> P_2$  in equilibrium. Taking this approximation in consideration, the ODE system can be writing as,

$$\left(\frac{dP_1}{dt} = -\left(\frac{C'}{2V_1}\right)P_1^2 + \left(\frac{Q_1}{V_1}\right)\right)$$

$$\left(\frac{dP_2}{dt} = \left(\frac{C'}{2V_2}\right)P_1^2 - \left(\frac{S_p}{V_2}\right)P_2$$
(3)

The solutions of this simplified system are,

$$P_{1} = -\frac{aV_{1}}{C'} \left( \frac{1 + C_{1}e^{at}}{1 - C_{1}e^{at}} \right)$$
(4)

and,

$$P_{2} = C_{2}e^{-bt} + 4\frac{b}{a}\frac{Q_{1}}{S_{p}}\left\{\left(\frac{1}{1+C_{1}}\right) + \frac{a}{4b} - \left(\frac{C_{1}}{1+C_{1}}\right)\frac{\left(e^{at} - e^{-at}\right)}{\left(C_{1}e^{at} - e^{-at}\right)} - \frac{b}{2}F_{1}\left[1, \frac{b}{a}; \left(1 + \frac{b}{a}\right); -C_{1}e^{2at}\right]\right\}$$
(5)

where the parameters  $a = [(C^{\circ}Q_1)/2]^{1/2}/V_1$  and  $b = S_p/(2V_2)$  are employed to simplify the equations presentation. The constants  $C_1$  and  $C_2$  are integration constants. The pressure in the processing chamber  $P_2$  is given in form of Hipergeometric Function [7], defined by,

$${}_{2}F_{1}(\alpha,\beta;\gamma;z) = \sum_{k=0}^{\infty} \frac{\alpha_{k}\beta_{k}}{\gamma_{k}} \frac{z^{k}}{k!}$$
(6)

where,  $\alpha_k = \alpha (\alpha + 1)... (\alpha + k)$ ,  $\beta_k = \beta (\beta + 1)... (\beta + k)$ ,  $\gamma_k = \gamma (\gamma + 1)... (\gamma + k)$ . The Hipergeometric Function has as constrain that |z| < 1 to assure it convergence. Because of that, the validity of the solution obtained using the equation (5) is limited to time intervals close to the initial conditions. To find the particular solution of equations (4) and (5), suitable to the experimental conditions found in laboratory it is enough to verify that when t = 0, then  $P_1(0) = P_2(0) = P_0$ , that is, the atmospheric pressure ( $P_0 = 101,303.25$  Pa). Then, the integration constants will be,

 $C_1 = \frac{1+s}{1-s} \tag{7}$ 

and,

$$C_{2} = P_{0} - 4\frac{b}{a}\frac{Q_{1}}{S_{p}}\left[\left(\frac{1}{1+C_{1}}\right) + \frac{a}{4b} - r\right]$$
(8)

where  $s = aV_1/C'P_0$  and  $r = {}_2F_1[1, (b/a); 1+(b/a); -(1+s)/(1-s)]$ . Replacing these values on equations (4) and (5),

$$P_1 = sP_0 \left[ \frac{1 + s \tanh(at/2)}{s + \tanh(at/2)} \right]$$
(9)

and,

$$P_{2} = P_{0}e^{-at} + 4\frac{b}{a}\frac{Q_{1}}{S_{p}}\left\{\left(\frac{1-s}{2}\right)\left(1-e^{-bt}\right) + \frac{a}{4b}\left(1-e^{-bt}\right) + re^{-bt} - (10) \\ \frac{\left(1-s^{2}\right)}{2}\times\left[1+s\frac{\left(e^{at}+e^{-at}\right)}{\left(e^{at}-e^{-at}\right)}\right]^{-1} - \frac{2F_{1}\left[1,\frac{b}{a};\left(1+\frac{b}{a}\right);-\left(\frac{1+s}{1-s}\right)e^{2at}\right]\right\}$$

The limitation related to the Hipergeometric Function convergence implies that  $|[(1+s)/(1-s)]e^{2at}| < 1$ . Thus the result expressed by equation (10) will be valid only for a small range of time, notably to the initials moments after the evacuation process begins. Nonetheless it equations can be used, with appropriate simplifications to gain more inside on the system behavior of the vacuum system after the steady-state condition be achieved, as will be explained below.

It is worthy to mention that the solutions represented by equations (9) and (10) are only valid to the pressure region where  $P_1 >> P_2$ . Thus, any results obtained from these equations to pressures close to atmospheric conditions are not representative.

# **3.2. NUMERIC SOLUTION**

One possibility to find the numeric solution of a differential equation is obtained by means of a Runge-Kutta method. This computational method departs from the differential equation itself and the respective initial conditions. Based on this information it is possible to calculate the slope of the curve that represents the answer of the problem. When the calculation is done, the final output is a series of coordinates that represents graphically the solution.

The employment of the Runge-Kutta method to a differential equation system is a little bit more evolving. In this case the equations must be solved simultaneously what, in a computational sense means that, on the calculation o each slope, information is needed on the current value of all the equations, not only on the equation the slope is been actually calculated. In the present problem, these complications leads to the calculation of two curves, representing the time variation of the pressure in each vessel. These curves were obtained based on the initial pressure that and on the differential equation itself. Then the slope of both curves was calculated until the final time value was achieved. At this point, the program ends and presents the final result. The numeric solution obtained using this method is compared to the analytic one in order to verify the accuracy of the approximations used above.

#### 3.3. STEADY STATE

In the vacuum system under study, the steady-state condition is achieved when the gas amount that enters a chamber equals the quantity that leaves it. On this situation, the throughput is constant through all the system. Considering the second chamber, that is, the reaction chamber, the throughput that leaves this vessel  $Q_{2out}$  is given by the product of the pressure inside this chamber and the pumping speed of the pump connected to it [1],

$$Q_{2out} = S_p \times P_2 \tag{11}$$

The gas that enters this chamber comes from the first chamber through the tube. In this case, the throughput that enters the chamber  $Q_{2in}$  is given by the very definition of a conductance [1],

$$Q_{2in} = C(P_1 - P_2) \tag{12}$$

As the system is in steady-state, it follows that the throughput of the gas entering the system must equals that of the gas leaving it. It means that  $Q_{2in} = Q_{2out} = Q_2$ . It leads to the equality,

$$S_{p}P_{2} = C(P_{1} - P_{2})$$
(13)

This expression can be further developed considering the Poiseuille-Hagen equation for long round tubes [1],

$$Q_2 = C' \frac{P_1 + P_2}{2} \left( P_1 - P_2 \right)$$
(14)

where, again,  $C^{c}$  is the amount of the tube conductance that doesn't change with the pressure, as defined in equation (2). Observing the definition of conductance expressed in (12), the conductance of a long round tube, in accordance with the equation above, can be written as,

$$C = C' \frac{P_1 + P_2}{2}$$
(15)

Combining equations (13) and (15) yields,

$$C'P_2^2 + 2S_p P_2 - C'P_1^2 = 0 (16)$$

The equation above is a second degree equation in  $P_2$ . It can be readily solved using the Bhaskara Formula to result in,

$$P_2 = \frac{-S_p + \sqrt{S_p^2 + C'^2 P_1^2}}{C'}$$
(17)

This result can also be expressed in the form,

$$P_{2} = \frac{S_{p}}{C'} \left[ \left( 1 + \frac{C'^{2} P_{1}^{2}}{S_{p}^{2}} \right)^{1/2} - 1 \right]$$
(18)

Expanding the square root term using the binomial equation and disregarding higher order terms as long as, in steadystate  $P_1 \ll 1$  and C' and  $S_p$  have above the same order of magnitude,

$$P_2 = \frac{C'}{2S_p} P_1^2 \tag{19}$$

Isolating  $P_1$  and considering equation (11),

$$P_1^2 = \frac{2}{C'} Q_2 \tag{20}$$

Now, remembering that in steady-state the throughput is equal everywhere, it must follows that  $Q_2 = Q_1$ , what, considering equation (12), means that the pressures on the chambers will be,

$$P_1 = \sqrt{\frac{2}{C'}\sqrt{Q_1}} \tag{21}$$

and,

$$P_2 = \frac{1}{S_p} Q_1 \tag{22}$$

This result can be confronted to the one obtained analytically. Considering equation (9), when  $t \to \infty$ , that is, in steady-state, tanh  $(at/2) \to 1$ . This result in,

$$P_1 \approx s P_0 \left(\frac{1+s}{s+1}\right) = s P_0 \tag{23}$$

Substituting the value of the parameters *s* and *a*,

$$P_1 \approx \frac{aV_1}{C'P_0} P_0 = \frac{aV_1}{C'} = \frac{\sqrt{2C'Q_1V_1}}{V_1C'}$$
 (24)

That is,

$$P_1 \approx \sqrt{\frac{2}{C'}} \sqrt{Q_1} \tag{25}$$

While in equation (10) as  $t \to \infty$ ,  $e^{-bt} \to 0$ ,  $e^{-at} \to 0$ ,  $e^{at} / e^{at} \to 1$ , and  ${}_2F_1[1, (b/a); 1+(b/a); -C_1 e^{-at}] \to 0$ , what implies that,

$$P_2 \approx 4 \frac{b}{a} \frac{Q_1}{S_p} \left[ \left( \frac{1-s}{2} \right) + \frac{a}{4b} - \left( \frac{1-s}{2} \right) \right]$$
(26)

That is,

$$P_2 \approx \frac{1}{S_p} Q_1 \tag{27}$$

That confirms the result obtained previously.

#### 4. RESULTS AND DISCUSSION

With the goal of compare the analytical and the numerical solutions, a particular system, with operational conditions similar to those find in practice was simulated. It was considered that the initial pressure in both chambers was the atmospheric pressure, that is, 760 Torr (101,323.25 Pa). This procedure simulates the process, commonly performed in laboratory of evacuate a plasma system, prior to normal operation. In this evaluation, the pump speed was supposed to be of 50 L/s, a reasonable value encountered in high vacuum systems. The chambers dimensions was those presented in section 2 of this article. In order to obtain a final pressure in the system it was considered that all secondary gas sources, as outgassing, leaks, thermal transpiration, etc. sum up to a constant throughput of 0.1 sccm ( $2x10^{-4}$  Pa.m<sup>3</sup>/s).



Figure 3 – Temporal variation of pressure in the two chambers. In blue is the pressure in the small volume and in green is the pressure in the reaction chamber.

Based on these constrains exposed above, two simulations were developed. The first considered the simplified system in differential equation (3); it is plotted on Figure 3. The second used the original set of equations in (1) and is show in Figure 4. As can be seen in the figures, both simulations resulted in the same steady-state. The first graph differs from the second only for the value of the small chamber pressure at the high pressure region. It happens because the first simulation consider a set o equation that was obtained taking  $P_1 >> P_2$ . This latter condition verifies only in the small pressure region. This interpretation is confirmed by the unreal condition seen in Figure 3, where, for the first seconds of evacuation, the pressure in the small chamber was less than the pressure in the process chamber. This behavior doesn't occurs in practice, and it appearance in this analysis reflect the weak assumption present in equation (3).



Figure 4 – Temporal variation of pressure in the two chambers. In blue is the pressure in the small volume and in green is the pressure in the reaction chamber.



Figure 5 – Pressure performance in the small chamber.

This analysis shows that the simplification adopted in equation (3) doesn't affect the final result concerning the steadystate condition, and is even irrelevant in respect to the process chamber solution. Thus the choice of what set of equation to simulate, at least in this case, was immaterial. It selection can be determinate by computational effort considerations, and of course, if is intended to obtain only the final value or if the hole pressure time excursion is sought. The Figures 5 and 6 compares the analytical and the numerical solutions directly. The blue solid line represents the numeric solution obtained as in Figures 3 and 4, using equation (1) for the numeric calculations, and the green solid line correspond to the analytical solution provided by equations (9) and (10), for the small chamber and the reactor chamber, respectively.



Figure 6 - Pressure performance in the reactor chamber.

The figures show that the results diverge most of the time (mainly in the small chamber), but in equilibrium conditions they are the same, irrespective the calculation was numeric or analytic. For Figure 6, the analytical result goes to infinity after some time. This behavior is connected to the nature of the Hipergeometric Functions that converge only in a small region where |z| < 1, as defined by equation (6). The equation (10) possesses a Hipergeometric Function with the variable t as an argument. When this argument becomes greater than unity the Hipergeometric Function diverges and, as a result, the graphic of  $P_2$  tends to infinity, as show in the figure below. This phenomenon resides not only on the variable t. It depends on the equations parameters s and a too. Theses parameters are related to the initial pressure  $P_0$ and the throughput  $Q_1$ , what means that the Hipergeometric Function will converge only for a small range of values of initial pressure and throughput. For the simulation presented in this article it was choose a specific value for this constants. The Figure 6 simply reflects a bad values choice.

Theses output from this analysis lead to the conclusion that the numerical calculation, despite not given a closed solution, provide the accurate response through all the conditions. The analytical calculation, on the other hand, is suited to final value calculations, as steady-state, and presents the advantage of supply a closed solution. This means that each approach is suitable in accord to the research needs.

The equations (21) and (22) were used to predict the value of the pressure on the small chamber and in the reactor chamber as a function of the gas throughput in the system. The result was compared to the measured values obtained in laboratory. The conclusion of this analysis, depicted in Figures 7 and 8 are quite satisfactory.



Figure 7 – Pressure in the small volume. In red solid line is represented the predictions of equation (21). The dots represent experimental measurements.

The experimental results adjusted well to the theoretical predictions. There is a small deviation only in the data from the reaction chamber pressure. This difference is caused by imperfections in the pressure gauge used to obtain the pressure in this chamber. The metering device was wore what result in a poor confidence in its measurements. Nonetheless the experiment shows great coherence with the theory.



Figure 8 – Pressure in the plasma chamber. In red solid line is represented the predictions of equation (22). The dots represent experimental measurements.

### 5. CONCLUSIONS

There are two aspects presented in this article. At first place it comprehends a discussion of the merits of the analytical approach versus the numeric calculations. Second, this paper presents a comparison between a theoretical analysis of a vacuum system and its respective experimental counterpart.

In respect to its first goal, this work shows that both, the analytical and the numerical solution has its own merits, and they are the best choice, depending on the researches needs. If the problem asks for a determination of a parameter value, in each point of the data range, then the numerical calculation is the best method. On the contrary, if only the steadystate solution is required and adequate approximations are aloud, then an analytical solution is enough, as long as the problem has a closed solution.

The second objective of this article consists in the verification of the adequacy of the theoretical analysis in predict the behavior of a real vacuum system. The experimental analysis in this article, due to technical limitations, was restricted to the steady-state condition. The results were in excellent agreement with the theory. Thus, the calculations done in this study succeed in describe the principle phenomena involved.

As a proposal to future studies, the author wishes to develop this analysis to investigate other characteristics of an experimental Hollow Cathode Plasma Etching System. This work will comprehend the determination of phenomena specific to orifice gas flow. By understanding its behavior it will be possible to investigate the gas-plasma interaction that occurs in an actual Hollow Cathode Plasma.

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